Closed trapped surfaces in higher dimensional Vaidya type solutions

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Abstract

In higher dimensional self-similar Vaidya spacetime and five dimensional ring type spacetime which has a self-similar mass function, we have constructed closed trapped surfaces which begin in a flat region, pass through a self-similar Vaidya region, and end in a black hole region, respectively. Moreover, we have shown that in both spacetimes, as long as spacetimes have closed trapped surfaces as above, a naked singularity never occur.

1 Introduction

A black hole is defined by an event horizon which is a boundary of a region in spacetime that cannot be observed from infinity. Therefore, unless we know an entire future evolution of spacetime, we cannot define the black hole. However, Eardley conjectured that the boundary of the region that contains marginally outer trapped surfaces coincide with the event horizon \cite{1}. So, a notion of outer trapped surfaces might be a definition of the black hole. For the four dimensional Vaidya solution, Ben-Dov shown that Eardley’s conjecture is true \cite{2}. It should be noted that outer trapped surfaces are difference from trapped surfaces as follows: outer trapped surfaces are closed spacelike $D-2$ surfaces whose outer null expansions is negative, on the other hand, trapped surfaces are closed spacelike $D-2$ surfaces whose both null expansions are negative.

In four dimensional Vaidya spacetime, if we consider trapped surfaces, spacetime has interesting features. Numerical results of Schnetter and Krishman shown that the outer boundary of trapped surfaces can extend into the flat region of Vaidya spacetime \cite{3}, and Bengtsson and Senovilla considered the self-similar Vaidya solution, and they constructed closed trapped surfaces that begin in a flat region, pass through a shell, and end in a Schwarzschild region \cite{4}. Hence, trapped surfaces are able to extend into the flat region. Moreover, Bengtsson and Senovilla shown that if spacetime has closed trapped surfaces as they constructed, a naked singularity never occur in this spacetime.

How are these features in higher dimensional black holes? Can we prove Eardley’s conjecture? and Do closed trapped surfaces extend into the flat region? At least, in higher dimensional Vaidya spacetime, if we are able to construct closed trapped surfaces that are similar to Bengtsson and Senovilla’s result by using the same manner to Ref. \cite{4}, we might be able to consider the definition of black holes as Eardley’s conjecture. Therefore, we concern both higher dimensional self-similar Vaidya spacetime and five dimensional ring type spacetime which has a self-similar mass function. Then we will construct closed trapped surfaces as Bengtsson and Senovilla’s result in both spacetimes.

In this paper, briefly we only consider higher dimensional self-similar Vaidya spacetime.

2 $D$ dimensional Vaidya spacetime

We consider a $D$ dimensional Vaidya solution \cite{5}

\begin{equation}
\begin{aligned}
ds^2 &= -\left(1 - \frac{2m}{(D-3)r^{D-3}}\right)dv^2 + 2dvdr + r^2 \left(d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \cdots + \sin^2 \theta_1 \cdots \sin^2 \theta_n d\theta_{n+1}^2\right),
\end{aligned}
\end{equation}

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which has a mass function given by

\[
m = \begin{cases} 
0, & v \leq 0 \\
\mu v^{D-3}, & 0 \leq v \leq M^\frac{1}{D-3}/\mu \\
M, & v \geq M^\frac{1}{D-3}/\mu
\end{cases}
\]

(2)

where \( n = D - 3 \), \( D \geq 4 \), \( \mu \) and \( M \) are constants, respectively. There is a radial influx of null fluid in an initially empty region of \( D \) dimensional Minkowski spacetime. For \( 0 \leq v \leq M^{1/(D-3)}/\mu \) spacetime is \( D \) dimensional Vaidya with the self-similar mass function, and for \( v \geq M^{1/(D-3)}/\mu \) we have \( D \) dimensional Schwarzschild spacetime. We know that a naked singularity will occur if and only if the mass function satisfies the following condition [6],

\[
\mu < \left( \frac{(D-3)}{2(D-2)} \right)^{D-2}.
\]

(3)

### 2.1 Two types of trapped surfaces

We consider two types of \( D - 2 \) surfaces as follows:

<table>
<thead>
<tr>
<th>Type 1</th>
<th>( \theta_1 = \frac{\pi}{2} ), ( r = R(\rho) ), ( v = V(\rho) ), ( \theta_2 = \phi_2 ), ( \cdots ), ( \theta_{n+1} = \phi_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta_2 = \frac{\pi}{2} ), ( r = R(\rho) ), ( v = V(\rho) ), ( \theta_1 = \phi_1 ), ( \theta_3 = \phi_3 ), ( \cdots ), ( \theta_{n+1} = \phi_{n+1} ) etc.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type 2</th>
<th>( \theta_1 = \Theta(\rho) ), ( v = V(\rho) ), ( r = \text{const} ), ( \theta_2 = \phi_2 ), ( \cdots ), ( \theta_{n+1} = \phi_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta_2 = \Theta(\rho) ), ( v = V(\rho) ), ( r = \text{const} ), ( \theta_1 = \phi_1 ), ( \theta_3 = \phi_3 ), ( \cdots ), ( \theta_{n+1} = \phi_{n+1} ) etc.</td>
</tr>
</tbody>
</table>

In Table 1, Type 1 surfaces follow that \( r \) and \( v \) are the function of \( \rho \), and the one of angles is constant. On the other hand, Type 2 surfaces obey that the one of angles and \( v \) are the function of \( \rho \), and \( r \) is constant.

By using these surfaces, we will construct closed trapped surfaces that begin in the flat region, pass through the self-similar Vaidya region, and end in the black hole region.

### 2.2 Closed trapped surfaces

Closed trapped surfaces are composed of the following parts:

- **flat region:** we consider type 1 surfaces and a topological disk given by the hyperboloid

\[
v = t_0 + r - \sqrt{r^2 + k^2}
\]

(4)

with constants \( t_0 \), \( k \), where \( t = v - r \) and \( R = \rho \).

- **self-similar Vaidya region:** we also consider type 1 surfaces and a topological cylinder defined by \( \theta = \pi/2 \) and

\[
\frac{dV}{dR} = \frac{a}{b - X}, \quad \text{where} \quad X = \frac{V}{R}, \quad \text{and} \quad R = \rho.
\]

(5)

where \( a, b \) are constants subject to \( a > b^2/4 \). Owing to this part, we will connect trapped surfaces in whole region.

- **black hole region:** we consider type 2 surfaces, and another disk composed of two parts

  - a cylinder with \( \theta = \pi/2 \), \( \gamma M = (D - 3)R^{D-3} \) where \( \gamma \) is a positive constant.
  - another final capping disk defined by

\[
\left( \Theta - \frac{\pi}{2} + \delta \right)^2 + \left( V \left( \frac{D-3}{\gamma M} \right)^{\frac{1}{D-3}} - \sigma_0 \right)^2 = \delta^2, \quad 0 < \delta \leq \frac{\pi}{2}.
\]

(6)

with constants \( \sigma_0 \) and \( \delta \).
In flat and self-similar Vaidya regions, all Type 1 surfaces are closed trapped surfaces, if and only if the mass function satisfies the following condition:

\[ \mu > \frac{1}{4} \left( \frac{(D - 3)}{\gamma} \right)^{\frac{1}{D - 3}}. \]  

(7)

In black hole region, the condition of trapped surfaces differ from each Type 2 surfaces. The surfaces, where \( v \) and \( \theta_1 \) are the function of \( \rho \), are trapped surfaces, if \( \gamma \) satisfies the following condition:

\[ (D - 3)\sqrt{\frac{2}{\gamma} - 1} \left( \frac{1}{\gamma} - 1 \right) > \frac{1}{\delta}. \]  

(8)

In Figure 1 we plot the upper bound of \( \gamma \) in Eq. (8) for dimensions by taking \( \delta = \pi/2 \). The solid line is the upper bound of \( \gamma \). It approaches to one for a infinitely large dimension.

Figure 1: The upper bound of \( \gamma \) in Eq. (8) for dimensions with \( \delta = \pi/2 \). The solid line is the upper bound of \( \gamma \). It approaches to one for a infinitely large dimension.

In Figure 1 we plot the upper bound of \( \gamma \) in Eq. (8) for dimensions by taking \( \delta = \pi/2 \). Since the upper bound of \( \gamma \) approaches to one for a infinitely large dimension, \( \gamma \) is less than one in any dimensions. Therefore, we do have closed trapped surfaces that begin in the higher dimensional Minkowski region and end in the higher dimensional Schwarzschild region.

Similarly, if \( \gamma \) satisfies the condition,

\[ (D - 3)\sqrt{\frac{2}{\gamma} - 1} \left( \frac{1}{\gamma} - 1 \right) > \frac{1}{\delta} \sin \theta_1, \]  

(9)

then the surfaces, where \( v \) and \( \theta_2 \) are the function of \( \rho \), are trapped surfaces. It should be noted that in the right hand side of Eq. (9) there is a factor \( \sin \theta_1 \), however, the right hand side of Eq. (8) does not depend on \( \sin \theta_1 \). Similarly, when we choose other surfaces and consider the condition of trapped
surfaces, the condition also has a factor of the angle dependence as the right hand side of Eq. (9). In Eq. (9), the condition for the upper bound of $\gamma$ depends on $\sin \theta_1$. If we take $\sin \theta_1 = 1$, then Eq. (9) is the same to Eq. (8). Besides, if $\sin \theta_1$ is negative, then the upper bounds of $\gamma$ become greater than one. However, since the trapping condition (9) is satisfied for any $\sin \theta_1$ in the region $\gamma < 1$, in this case we also do have closed trapped surfaces. Although the condition of trapped surfaces depends on the choice of the $D-2$ surfaces, all conditions are satisfied in the region $\gamma < 1$. Therefore, we do have obtained the closed trapped surfaces in higher dimensional self-similar Vaidya spacetime.

Here, let us consider about the naked singularity in this spacetime. In Type 2, by substituting Eq. (8) into Eq. (7) we can take the mass function which depends on dimensions. If spacetime satisfies both conditions Eqs. (7) and (3), i.e. if $\gamma$ satisfies the condition

$$\gamma > (D - 3) \left[ \frac{1}{4} \left( \frac{2(D - 2)}{(D - 3)} \right) \right]^{D-3},$$

(10)

then spacetime has the naked singularity. However, Eq (10) does not satisfy $\gamma < 1$. Therefore, in our discussion, if spacetime has closed trapped surfaces given by above discussions, then it never has the naked singularity.

### 3 Conclusion

We have considered both higher dimensional self-similar Vaidya spacetime and five dimensional ring type spacetime which has the self-similar mass function. In both spacetimes, we have utilized the same manner to Ref. [4] , and then we have shown the following results. First, in both spacetimes, although conditions of the trapped surface depend on the choice of the $D-2$ surfaces, that conditions are satisfied in the region as $\gamma < 1$. Therefore, we do have constructed the closed trapped surfaces, respectively: that begin in the flat region, pass through the self-similar Vaidya region, and end in the black hole region. Second, to notice the constant of the mass function $\mu$ we have shown that the naked singularity never occur in both spacetimes with the closed trapped surface given by our discussions.

These results are similar to that in four dimensional self-similar Vaidya spacetime. Hence, we might be able to consider the definition of black holes by using Eardley’s conjecture in higher dimensional spacetime.

### References