Constraining alternative theories of gravity with space-borne gravitational wave interferometers

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Abstract

We calculate the possible constraints we can put on alternative theories of gravity such as Brans-Dicke and massive gravity theories by gravitational waves from inspiral compact binaries. We take both precession and small eccentricity into account for the first time. We perform Monte Carlo simulations and by using the Fisher analysis, we estimate the determination accuracy of binary parameters including the Brans-Dicke parameter and the graviton Compton wavelength. By using LISA, although the constraint on Brans-Dicke theory is several times weaker than the current strongest one, the constraint on the mass of graviton is four orders of magnitude stronger than the one obtained by the solar system experiment. With DECIGO, the former constraint increases considerably.

1 Introduction

One of the approaches to solve dark energy problem is to modify gravitational theory from general relativity. In this paper, we consider two simple modifications of gravity. One is to add scalar degree of freedom to gravity. This theory is called scalar-tensor theory \cite{1}. This theory also appears in inflation problem and superstring theory. A prototype of scalar-tensor theory is Brans-Dicke theory \cite{2}. This theory is characterised by a parameter $\omega_{BD}$ and by taking the limit $\omega_{BD} \rightarrow 1$, it reduces to general relativity. The current strongest bound on $\omega_{BD}$ is the Cassini bound obtained from the solar system experiment \cite{3}; $\omega_{BD,\text{Cassini}} > 40000$.

Another type of modification of gravity introduces a finite mass $m_g$ to a graviton (see \cite{4} for a recent review). This type of theory is called the massive gravity theory. From the solar system experiment, the constraint on graviton Compton wavelength $\lambda_g$ (which is defined as $\lambda_g \equiv h/m_g c$) has been obtained, by using Kepler’s third law, as $\lambda_g > 2.8 \times 10^{17}$ cm \cite{5}.

The aim of our work is to investigate how strongly we can constrain $\omega_{BD}$ in the strong field regime by detecting gravitational waves from compact object binaries. In Brans-Dicke theory, the additional scalar field contains the dipole radiation \cite{6,7}. This modifies the binary’s orbital evolution from the one in general relativity. The change in the orbital evolution due to this dipole radiation modifies the phasing of the gravitational waveform. In massive gravity theories, the propagation speed of gravitational wave depends on its frequency, which modifies the time of arrival from general relativity. This also affects the phasing of the gravitational waveforms \cite{8}. Recently, Berti \textit{et al.} \cite{9} estimated how accurately one can determine these additional parameters by using space interferometer LISA. We extend their work by including two important effects, precessions and eccentricities. We also calculate the constraints in the case of using DECIGO.

2 Waveforms

For the waveforms, we use the restricted 2PN waveforms. The Fourier component of binary gravitational waveform in Brans-Dicke theory or massive gravity theory is given by \cite{10}

$$\tilde{h}(f) = \frac{\sqrt{3}}{2} \frac{5}{4} A^7 c^{7/6} e^{i\Psi(f)} \left[ A_{\text{pol},\alpha}(t(f)) \right] e^{-i\left(\varphi_{\text{pol},\alpha} + \varphi_{\text{D}}\right)},$$

(1)
where the amplitude $A$, the polarisation amplitude $A_{\text{pol,} \alpha}^{\text{prec}}$, the polarisation phase $\varphi_{\text{pol,} \alpha}^{\text{prec}}$, and the Doppler phase $\varphi_{D}$ are defined in Ref. [10]. The phase $\Psi(f)$ is given by [10]

$$
\Psi(f) = 2\pi f_{c} - \phi_{c} - \frac{\pi}{4} + \frac{3}{128} (\pi M f)^{-5/3} \left[ 1 - \frac{355}{1462} I_{c} f^{-19/9} - \frac{5}{84} S^{2} \bar{\omega}_{x} x^{-1} \right. \\
\left. - \frac{128}{3} \beta_{g} \eta^{2/3} x + \left( \frac{3715}{756} + \frac{55}{9} \right) (4 \pi - \beta) x^{3/2} \right. \\
\left. + \left( \frac{15293365}{508032} + \frac{27145}{504} \eta + \frac{3085}{72} \eta^{2} - 10 \sigma \right) x^2 \right].
$$

(2)

Here, $f_{c}$ and $\phi_{c}$ are the coalescence time and phase, respectively. $M \equiv \mu^{3/5} M^{2/5}$ is the chirp mass with $M$ being the total mass and $\mu$ being the reduced mass, and $\eta \equiv M/\mu$ is the symmetric mass ratio. We defined the squared typical velocity, $x \equiv v_{\text{typ}}^{2} = (\pi M f)^{2/3}$. The first term in the square brackets represents the lowest order quadrupole approximation of general relativity. The second term is the contribution from small eccentricity. $I_{c}$ is the asymptotic eccentricity invariant defined in Ref. [11]. The third term represents the dipole gravitational radiation in Brans-Dicke theory. $\bar{\omega} \equiv \omega_{\text{BD}}^{-1}$ is the inverse of the Brans-Dicke parameter. $S \equiv s_{2} - s_{1}$ with $s_{i}$, the so-called sensitivity, is defined in Ref. [9, 10]. This roughly equals to the binding energy of the body per unit mass. Binaries with large $S$ are the ones composed of bodies of different types.

The fourth term is the contribution from the mass of graviton. When graviton is massive, the propagation speed is slower than the speed of light, which modifies the gravitational wave phase from general relativity. $\beta_{g}$ is defined as [9, 10] $\beta_{g} \equiv \pi^{2} D M / \lambda_{5}^{2} (1 + \frac{z}{3})$, where $z$ is the cosmological redshift and the distance $D$ is defined in Ref. [9, 10]. The remaining terms are the usual higher order PN terms in general relativity, where $\beta$ and $\sigma$ are spin-orbit coupling and spin-spin coupling, respectively.

Since the polarisation amplitude $A_{\text{pol,} \alpha}^{\text{prec}}$ and the polarisation phase $\varphi_{\text{pol,} \alpha}^{\text{prec}}$ depend on the direction of the orbital angular momentum $\mathbf{L}$, precession affects the waveform through these two quantities. For simplicity, we assume that one of the spins of the binary constituents is negligible. Under this so-called simple precession approximation, the precession equations can be solved analytically [12, 13].

### 3 Numerical Calculations and Results

We perform the Monte Carlo simulation and estimate the statistical determination accuracies $\Delta \theta^{\alpha}$ of binary parameters $\theta^{\alpha}$ by using Fisher analysis. $\Delta \theta^{\alpha}$ can be calculated as $\Delta \theta^{\alpha} = \sqrt{\Sigma^{\alpha \alpha}}$, where the covariance matrix $\Sigma^{\alpha \beta}$ is the inverse of the Fisher matrix $\Gamma_{\alpha \beta} \equiv \langle \partial_{\theta^{\alpha}} \partial_{\theta^{\beta}} \rangle$. Here, $\partial_{\theta}$ represents the derivative with respect to $\theta$, and the inner product $(A|B)$ is defined in Ref. [10].

For Brans-Dicke theory, we consider $(1.4+10^{3}) M_{\odot}$ NS/BH binaries for LISA and $(1.4+10) M_{\odot}$ NS/BH binaries for DECIGO. We assume that the signal to noise ratio (SNR) is $\sqrt{200}$ and the difference between NS and BH sensitivities $S$ is 0.3. For massive gravity case, we think of $(10^{7}+10^{6}) M_{\odot}$ BH/BH binaries for LISA and $(10^{9}+10^{8}) M_{\odot}$ BH/BH binaries for DECIGO. We fix the distances of these binaries to 3 Gpc in this case. We assume that observation starts 1 year before coalescence.

There are 15 parameters, all of which are parametric in Ref. [10]. Out of these, we are particularly interested in the determination accuracies of $\bar{\omega}$ and $\beta_{g}$. To perform Monte Carlo simulations, we randomly generate the following 6 quantities: the inner product of the orbital angular momentum and the total spin angular momentum $\kappa$; the precession angle $\alpha_{c}$ (which is defined in Ref. [10]); $\theta_{s}, \phi_{S}$ for the initial direction of the source; $\theta_{1}, \phi_{1}$ for the initial direction of the total angular momentum. We calculate the parameter estimation errors for each binary and take the average.

We found that inclusion of eccentricity weakens the constraints. This is because the parameters are strongly correlated and adding parameters dilutes the binary information in the detected gravitational waves. However, when we include the prior information of $\Delta I_{c} > 0$, we found that the constraint on $\omega_{\text{BD}}$ becomes the same as the one without including eccentricity into binary parameters. For massive gravity case, the effect of eccentricity is weaker compared to Brans-Dicke case. This is because the frequency dependence of the eccentricity, Brans-Dicke and massive gravity terms in the phase $\Psi(f)$ (Eq. (2)) is $f^{-19/9}$, $f^{-2/3}$ and $f^{2/3}$, respectively, so that eccentricity has more degeneracy with $\omega_{\text{BD}}$ than with $\lambda_{g}$. 


Figure 1: (Left) The histograms showing the probability distribution of the lower bound of $\omega_{BD}$ obtained from our Monte Carlo simulations of $10^4$ NS/BH binaries in Brans-Dicke theory. We take the masses of the binaries as $(1.4 + 10^3)M_{\odot}$ for LISA and $(1.4 + 10)M_{\odot}$ for DECIGO with SNR = $\sqrt{200}$. The (light blue) dotted-dashed histogram shows the constraint without precession and the (blue) dashed one represents the one including precession. The (purple) dotted one represents the estimate without precession and the (red) solid one shows the one including precession using DECIGO. The (green) dashed line at $\omega_{BD} = 4 \times 10^4$ represents the Cassini bound [3]. (Right) The histograms showing the probability distribution of the lower bound of $\lambda_g$ obtained from our Monte Carlo simulations of $10^4$ BH/BH binaries in massive gravity theories. We take the masses of the binaries as $(10^7 + 10^8)M_{\odot}$ for LISA and $(10^6 + 10^7)M_{\odot}$ for DECIGO at 3Gpc.

In Fig. 1, we show the probability distribution of $\omega_{BD}$ (in the left panel) and $\lambda_g$ (in the right panel) with and without including precession whilst eccentricity is not included into binary parameters. The (light blue) dotted-dashed histogram shows the constraint without precession and the (blue) dashed one represents the one including precession. The (purple) dotted one represents the estimate without precession and the (red) solid one shows the one including precession using DECIGO. The (green) dashed line at $\omega_{BD} = 4 \times 10^4$ in the left panel represents the Cassini bound [3]. From these results it can be seen that the effect of precession makes the constraints stronger. This is due to the fact that precession disentangles the degeneracies between binary parameters. We found that for LISA, the constraint on $\omega_{BD}$ is several times weaker than the Cassini bound, whilst the constraint is 200 times stronger than the Cassini bound when we use DECIGO. This is mainly because the number of gravitational wave cycles is larger for DECIGO and also because the noise levels of DECIGO are lower than that of LISA. For the massive gravity case, the constraint on $\lambda_g$ is four orders of magnitude stronger than the one from solar system experiment when we use LISA. In this case, the constraint obtained by DECIGO is slightly weaker than the one with LISA. This is because the masses of binaries are larger for LISA.

4 Conclusions

We estimated how strongly we can put constraints on $\omega_{BD}$ and $\lambda_g$ by detecting gravitational waves from inspiralling compact binaries using LISA and DECIGO. We included the effects of eccentricity and precession for the first time. In order to estimate the constraints, we performed following Monte Carlo simulations. We randomly distribute $10^4$ binaries all over the sky, evaluate the parameter estimation accuracies for each binary, and take the average. We found that inclusion of eccentricity makes the constraints weaker but the effect of precession enhances the constraints. For the Brans-Dicke case, by using NS/BH binaries with SNR = $\sqrt{200}$, the constraint by LISA is weaker compared to the Cassini bound.
whilst DECIGO can put 200 times stronger constraint than the Cassini one. Unlike the case of LISA, these binaries are thought to be the definite sources for DECIGO. The event rate of NS/NS binary mergers is estimated to be $10^5 \text{yr}^{-1}$ [14], and the rate of NS/BH mergers will be about one order of magnitude smaller than that of NS/NS mergers (see Shibata et al. [15] and references therein). Therefore it is possible to put even stronger constraint by performing a statistical analysis. In this case, we found that the constraint becomes $\omega_{\text{BD}} > 5.74 \times 10^7$. This is three orders of magnitude stronger than the Cassini bound.

For massive gravity case, by using BH/BH binaries at 3 Gpc, LISA can put four orders of magnitude stronger constraint than the one obtained by the solar system experiment. We found that the constraint with DECIGO is slightly weaker than the one with LISA. Please see Refs. [10, 16] for more details.

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