Gravitational waves and Q-ball formation

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Abstract

The detectability of the gravitational waves (GWs) from the Q-ball formation associated with the Affleck-Dine (AD) mechanism is studied. We take into account both of the dilution effect due to Q-ball domination and of finite temperature effects. We find that there is a finite but small parameter region where such GWs may be detected by future detectors such as DECIGO or BBO, only in the case when the thermal logarithmic potential dominates the potential of the AD field. Unfortunately, for such parameter region the present baryon asymmetry of the universe can hardly be explained unless one fine-tunes A-terms in the potential.

1 Introduction

Primordial gravitational waves (GWs) provide us with a lot of important information of the early universe. Recently, a new interesting mechanism to produce GWs was proposed in Refs. [1, 2], in which it is shown that significant GWs are generated during the Q-ball formation associated with Affleck-Dine (AD) mechanism of baryogenesis [3]. The AD mechanism implies the formation of non-topological solitons, Q-balls, whose existence and stability are guaranteed by a conserved charge, $Q$ [4]. The Q-balls are formed by the amplification of fluctuation around homogeneous scalar fields that carry baryon or lepton number. Since this formation process of such Q-balls is inhomogeneous and not spherical, GWs can be generated during the formation.

In order to calculate the present properties of such GWs, one has to not only estimate the amount of GWs at the formation of Q-balls but also take into account the cosmic history after the production of the GWs. The energy density of the GWs at the Q-ball formation is proportional to some powers of the field value of the AD condensate, which implies that the initial energy density of the GWs becomes large if the typical charge $Q$ of Q-balls is large. On the other hand, the lifetime of Q-balls becomes longer for larger $Q$ because the temperature at the decay of Q-balls is typically proportional to the inverse square-root of the charge $Q$. Therefore, Q-balls with large $Q$ can quickly dominate the energy density of the universe and hence dilute the GWs significantly. Thus, the detectability of such GWs is determined by the balance of the above two competing effects. In this study, we consider the decay of Q-balls without exotic effects by which Q-balls decay quickly and calculate the amplitude and frequency of GWs from Q-balls taking into account of the dilution factor correctly, which results in dramatic changes in the present amplitude of the GWs from the Q-ball formation.

The properties of the Q-balls depend on shape of the effective potential, which varies with the supersymmetry (SUSY) breaking mechanism and other effects. Thus, there are many types of Q-balls. Among them, thermal log type Q-balls that are formed by thermal (logarithmic) effects have an interesting feature. The energy density of this type of Q-balls decreases at least as rapid as radiation. This is because the thermal logarithmic potential itself also decreases with the cosmic expansion while the number of Q-balls in a comoving volume does not change. Hence, this type of Q-balls cannot dominate the energy density of the universe and do not dilute GWs. This is favorable for the detection of the

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GWs from the Q-ball formation because the dilution during the Q-ball dominated era is the main obstacle for the detection. Therefore, in this study, we concentrate on the thermal log type Q-balls and estimate the present amplitudes and frequencies of the GWs at the formation of such Q-balls. We show that such GWs may be detected by the next-generation gravitational detectors like DECIGO and BBO if particular conditions of reheating temperature, the initial field value of the AD field, gravitino mass and messenger mass are realized in the gauge mediated SUSY breaking model. However, we also find that such a condition spans a very small region in the parameter space. Moreover, we also find that it is difficult to explain the present baryon asymmetry for such a parameter region unless one fine tunes the CP-violating A-terms in the potential.

2 Affleck-Dine mechanism and Q-ball

In supersymmetric theories, there are many flat directions along which the scalar potentials become flat in the global SUSY limit and some of them carry baryon and/or lepton number. In the realistic world, SUSY is broken and the potentials for the flat directions are slightly lifted, the way of which depends on the SUSY breaking mechanism. Moreover, in the presence of thermal plasma, it also acquires thermal corrections. Thus, scalar fields that moves along a flat direction feels a force from the potentials. Their dynamics can be expressed in terms of a scalar field Φ (AD field). Hereafter we only consider the dynamics of a AD field $\Phi = e^{i\theta}/\sqrt{2}$.

In the early universe, the AD field can acquire large field values. Then it starts to rotate around the origin when the Hubble parameter reaches its effective mass. The angular velocity is provided by CP-violating A-terms from the SUSY breaking effects. If the AD field carries baryon or lepton charge $c$, the angular momentum of the motion in the complex plane of the AD field represents the baryon or lepton number density given by

$$n_B(t_{osc}) = i\beta_e(\dot{\Phi}^*\Phi - \Phi^*\dot{\Phi}) \simeq \beta_e a_m m_{3/2}\phi_{osc}^2,$$

which implies that baryon or lepton asymmetry is generated in the universe. Here $a_m$ is a CP-violating parameter of the order of unity in general, $m_{3/2}$ is the gravitino mass and $\phi_{osc} = |\Phi_{osc}|/\sqrt{2}$ is the amplitude of the AD field at the onset of the rotation around the origin.

Next we consider the Q-ball formation. Fluctuations around the homogeneous mode feel spatial instabilities and grow nonlinearly during the oscillation of the AD field and eventually form clumpy objects, Q-balls, if $V(\phi)/\phi^2$ has a global minimum at $\phi = \phi_{min} \neq 0$. In many cases, Q-balls are formed just after the onset of the rotation of the AD field.

The way of amplification of fluctuations and the properties of Q-balls are different with the potential for the AD field. When thermal logarithmic potential,

$$V_{thermal} \simeq a_3^2T^4 \log \left( \frac{|\Phi|^2}{T^2} \right),$$

dominates the potential for the AD field, these thermal log type Q-balls have an interesting character. Since the dominant contribution to the potential depends on the temperature which changes with the cosmic time, the properties of Q-balls change as well. Moreover, for some temperatures, other contributions can dominate the thermal logarithmic contribution in the potential, which implies that the properties of Q-balls may drastically change and Q-balls may disappear if the dominant contribution of the potential does not allow a Q-ball solution. In the gauge mediated SUSY breaking mechanism, for example, the potentials that dominate the potential for the AD field afterwards should be

$$V_{grav} = n_{3/2}^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M_F^2} \right) \right] |\Phi|^2 \text{ and } V_{gauge} = M_F^2 \left( \log \frac{|\Phi|^2}{M_F^2} \right)^2.$$

Here $M_F$ is the messenger mass in the gauge mediated SUSY breaking models. When $V_{grav}$ with positive $K$ dominates first, the fate of Q-balls is rather complicated. This potential does not allow a Q-ball formation even before the reheating from inflaton decay, because its partial decay products are easily thermalized. 

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3Thermal plasma can exist even before the reheating from inflaton decay, because its partial decay products are easily thermalized.
solution and hence the almost homogeneous AD field is recovered. After this transition, the value of the AD field decreases due to the cosmic expansion. Thus, $V_{\text{gauge}}$ dominates $V_{\text{grav}}$ gradually, which implies that Q-balls are formed again.

Q-balls or the homogeneous AD field can dominate the energy density of the universe from when $V_{\text{grav}}$ dominates the potential. Thus, it is important to estimate the time when they dominate the energy density of the universe and their decay rate. The Q-ball domination time can be evaluated by the energy density of the Q-balls at the Q-ball formation and the Q-ball transformation time. In the case described above, the Hubble parameter at the Q-ball domination is given by

$$H_{\text{dom}} \simeq 10^{-3} \frac{m_{3/2}^2 \phi_{\text{osc}}^8}{M_G T_R^2},$$

where $M_G$ is the reduced Planck mass and $T_R$ is the reheating temperature.

Then we consider the decay rate of Q-balls. Q-balls can decay into light fermions if the decay processes are kinematically allowed. However, in their interiors the Pauli exclusion principle forbids their decays into fermions [6]. Therefore Q-balls can decay only from their surfaces. This sets the upper bound on the decay rate of Q-balls, and in fact, it is almost saturated for the cases we are interested in [6]. In the case described above, the Hubble parameter at the Q-ball domination is given by

$$H_{\text{dec}} \simeq 0.3 \times \frac{m_{3/2}^2}{M_F^2}.$$  

### 3 Gravitational waves from Q-ball formation

Now we consider the generation of the GWs. As mentioned before, the process of formation of Q-balls is inhomogeneous and not spherical. Thus, GWs are emitted at that time. In the case of the thermal log type Q-balls, the density parameter of the GWs from Q-balls and their typical frequency are given by [5]

$$\Omega_{\text{GW}}^* \simeq 2 \times 10^{-6} \left( \frac{\phi_{\text{osc}}}{M_G} \right)^4, \quad \text{and} \quad f_* \simeq 0.04 \times \frac{T_{\text{osc}}^2}{\phi_{\text{osc}}}.$$  

The energy density of the GWs can be rather large if $\phi_{\text{osc}}$ is large.

The density parameter of GWs are diluted during the inflaton oscillation dominated era, the Q-ball dominated era and recent matter and dark energy dominated era, and their frequency is also redshifted by cosmic expansion. Thus, the present properties of the GWs from the Q-ball formation are given by [5],

$$\Omega_{\text{GW}}^0 = \Omega_{\text{GW}}^* \times \begin{cases} \left( \frac{H_R}{H_*} \right)^{2/3} \left( \frac{H_{\text{dec}}}{H_{\text{dom}}} \right)^{2/3} \frac{a_{\text{eq}}}{a_0} & \text{(with Q - ball domination),} \\ \left( \frac{H_R}{H_*} \right)^{2/3} \frac{a_{\text{eq}}}{a_0} & \text{(without Q - ball domination),} \end{cases}$$

$$f_0 \simeq f_* \times \begin{cases} \left( \frac{T_0}{T_R} \right)^{1/6} \left( \frac{H_R}{H_*} \right)^{2/3} & \text{(with Q - ball domination),} \\ \left( \frac{T_0}{T_R} \right)^{2/3} & \text{(without Q - ball domination).} \end{cases}$$

Here $T_0$ is temperature at present and $a_{\text{eq}}$ and $a_0$ are the scale factors at the matter-radiation equality and at present, respectively.

When in the gauge mediated SUSY breaking mechanism with $V_{\text{grav}}(K > 0)$, we find that when

$$M_F \simeq 10^4 \text{GeV}, \quad m_{3/2} \simeq 10\text{GeV}, \quad \phi_{\text{osc}} \simeq M_G \quad \text{and} \quad T_R \simeq 10^{16}\text{GeV},$$

there is Q-ball dominated era and we have $\Omega_{\text{GW}} \simeq 10^{-16}$ for $f_0 \simeq 10\text{[Hz]}$ [5]. This is on the edge of the DECIGO or BBO sensitivity range. Thus, it is difficult but not impossible to detect such GWs by the
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next generation detectors. Moreover there is no other parameter set to realize GW emissions suitable for detection [5]. One should notice that though gravitino does not overclose the universe because of large entropy production from Q-ball decay, the next-to-lightest supersymmetric particle (NLSP) decay may spoil the success of the BBN in some cases since the hadronic decay product of NLSP would destroy the light elements.

Here we comment on the present baryon and lepton asymmetry. In the situation where the GWs from the Q-ball formation might be detected, both present baryon/lepton asymmetry and radiation are generated by the Q-ball decay. The present baryon/lepton-to-entropy ratio would be rather large and be of the order of unity unless the parameter in the A-term, $m_a$, is strongly suppressed. In the case of Q-ball with baryonic charge, it is far beyond the experimental bound on the present baryon asymmetry. Even in the case of L-balls, that is, Q-balls with lepton charge but without baryon charge, the situation does not change since the decay temperature of Q-ball is about 500 GeV and hence the lepton asymmetry is converted to baryon asymmetry by sphaleron process that conserves $B - L$ charge. The way to avoid such large baryon/lepton asymmetry in this scenario is that the Q-balls are made of the AD field with $B - L = 0$. In this case, however, we need other baryogenesis mechanisms.

4 Conclusion

In this study, we have discussed the detectability of the GWs from the Q-ball formation. At the Q-ball formation, Q-balls with large $Q$ can produce a large amount of GWs. However, such Q-balls decay slowly and they may dominate the energy density of the universe so that GWs are significantly diluted. Therefore the detectability of the GWs is determined by these two competing effects.

We have shown that in the gauge mediated SUSY breaking model, if the reheating temperature is $T_R \sim 10^{10}$ GeV and the initial field value of the AD field is $\phi_{osc} \sim M_G$ with $m_{3/2} \sim 10$ GeV and $M_F \sim 10^4$ GeV, the present density parameter of the GWs from the Q-ball formation can be as large as $\Omega_{GW}^0 \sim 10^{-16}$ and their frequency is $f_0 \sim 10$ Hz. Thus, it is difficult but not impossible to detect them by next-generation gravitational detectors like DECIGO or BBO, but the parameter region for detectable GWs is very small. In other cases, it is shown that it is almost impossible to detect GWs from Q-ball formation [5].

Moreover, there are difficulties in this successful parameter region. One is that such parameter region predicts too large baryon asymmetry. Thus, once the GWs from the Q-ball formation are detected, we have the following two possibilities. In the case that such Q-balls are responsible for the present baryon asymmetry, the A-terms are suppressed by symmetry reason. The second option is that Q-balls are irrelevant for baryogenesis, which is realized for the AD fields with $B - L = 0$. Another is the identification of such GWs. A first order phase transition in the early universe would produce similar spectrum of GWs. However, in our case, the gravitino mass must be around 10 GeV for the detection of the GWs from the Q-ball formation. Thus, if collider experiments could determine the gravitino mass by measuring the lifetime of the NLSP, that would provide complemental information or even rule out this scenario.

References