Interior of a charged black hole with an exotic scalar field

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Abstract

We use a numerical code to investigate the nonlinear processes arising when a Reissner-Nordström black hole (RNBH) is irradiated by an exotic scalar field beam. These processes are quite different from the processes arising in the case of the same black hole being irradiated by a beam of a normal scalar field. For full results and detailed analyses, see \cite{1}.

1 Introduction

The internal structure and physics of black holes (BH) has been the subject of researches during many years. A powerful tool for such investigations is to consider a Reissner-Nordström BH (RNBH) which is nonlinearly perturbed by a selfgravitating scalar field. This toy BH model is not very realistic but it shares many properties, including causal structure, with the more realistic rotating BHs.

This toy model has been used in the paper \cite{2} to analyze the physics of the interior of a BH in the case of irradiation by a normal massless scalar field. Recent astrophysical observations suggests that a considerable part of the matter in the Universe consists of a hypothetical dark energy, exotic matter, which violates at least the strong and perhaps also the weak energy condition. These discoveries and other theoretical investigations put a question about the physics of the interior of a BH nonlinearly perturbed by exotic matter.

The goal of this work is to perform such an analysis by using numerical methods. Our numerical code was described and tested in details in \cite{1,3}. We will see that the physics of the interior of a BH with an exotic scalar field is quite different from the physics of a BH with a normal scalar field.

Due to space requirements, only a summary of results can be presented in this text. For full discussions and complete results, please see \cite{1}.

2 Our model

We investigate the evolution of a spherical BH with a fixed electric charge \(q = 0.95m0\) (i.e. Reissner-Nordström metric) and initial mass \(m0 = 1\), which is under the action of pulses of an exotic scalar field \(\Psi\) (modelled as a massless, selfgravitating scalar field with a negative kinetic energy term, i.e. it has a negative energy density, \(\varepsilon < 0\) \cite{1,3}).

For our model and numerical analysis, we use double null coordinates. The line element in double null coordinates can be written as

\[
ds^2 = -2\varepsilon^{2\sigma(u,v)}du dv + r^2(u,v)d\Omega^2, \tag{1}
\]

where \(\sigma(u,v)\) and \(r(u,v)\) are functions of the null coordinates \(u\) and \(v\) (in- and out-going respectively).

The energy-momentum tensor can be written as a sum of contributions from the exotic scalar field \(\Psi\) and from the ordinary magnetic field, i.e.: \(T_{\mu\nu} = T^\Psi_{\mu\nu} + T^{em}_{\mu\nu}\).

The full non-linear Einstein equations in this case become rather simple and are easily written out in their complete form. They reduce to a set of evolution equations which are supplemented by two constraint equations (for the full explicit expressions, see \cite{1}).

We wish to numerically evolve the unknown functions \(r(u,v), \sigma(u,v)\) and \(\Psi(u,v)\) throughout some computational domain. We do this by following the approach described in \cite{2,3} (and references therein).

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to numerically integrate the evolution equations. These equations form a well-posed initial value problem in which we can specify initial values of the unknown functions on two initial null segments, namely an ingoing ($v = v_0 = constant$) and an outgoing ($u = u_0 = constant$) segment. We impose the constraint equations on the initial segments, consistency of the evolving fields with the constraint equations is then ensured via the contracted Bianchi identities. However we use the constraint equations throughout the domain of integration to check the accuracy of the numerical simulation.

Our choice of initial values corresponds to the following physical situation; There is a Reissner-Nordström BH and at some distance from the horizon, at the initial moment there is a rather narrow spherical layer of an in-falling exotic scalar field (see [1] for details).

**3 The case of survival of the BH (weak pulse)**

We start from the case when the power of the exotic beam is rather weak. We shall see that in this case, the BH survives but the positions of the horizons change.

In fig. 1(a) is shown the causal structure for the case of weak pulse, including location of $R$ and $T$ regions and positions of the horizons. As was shown in [2], sending a pulse of the normal scalar field into a RNBH causes the positions of the horizons change, the outer apparent horizont (OAH) in that case will go to larger $r$ and the inner apparent horizon (IAH1) will go to smaller $r$. In fig. 1(a), we see that the opposite effect takes place when the pulse is an exotic scalar field. I.e. instead of diverging, the two horizons (OAH and IAH1) now converge towards each other. This is a direct effect of the negative energy content of the exotic scalar field. However, the two horizons do not meet and thus the BH is not destroyed.

Fig. 2(a) shows the mass function, i.e. the total mass (without the magnetic field) in a sphere of radius $r(u, v)$ (see [2] and refs. therein). It is seen that for small $v$, when the pulse enters the RNBH (between $5 \leq v \leq 9$), the mass function decreases as a direct result of the negative mass being sent into the BH. This is also opposite to the effect of a positive energy pulse[2]. Soon thereafter, however, the mass function increases for those lines of constant $u$ which are inside of the BH (for $u \geq 24$). This increase of the mass function is partly related to scattering of the exotic field, partly related to the mass inflation effect (see [5]), which also works in the case of an exotic scalar field. The mass function along $u = 0$ (border of our computational domain) is seen to be smaller than 0.95 which would normally indicate that the RNBH (with charge $q = 0.95m0$) would vanish. However, because of scattering effects, not all the radiation from $u = 0$ reaches the BH (around $u = 23$), but is scattered away due to the curvature of the
space-time. Thus, the mass near the OAH is “only” reduced to approx. 0.96, which (in agreement with fig. 1(a)), indicated that the BH survives the pulse of exotic radiation.

Our analysis allows us to guess at the Penrose diagram for the case when a pulse of exotic scalar radiation is sent into a RNBH which is weak enough that the BH survives the pulse. This Penrose diagram can be seen in fig. 3(a).

4 The case of destruction of the BH (strong pulse)

Next we consider a pulse which has a higher negative energy. Fig. 1(b) shows the further evolutions of the general picture of the R and T regions for this case.

Now, total power of the exotic beam is large enough to reduce the mass of the object in the critical region $u \approx [23.2; 23.9]$ (where the OAH formed for the previous case), to below the critical value $m_{crit} = q = 0.95$, see Fig. 2(b). Thus even after the reduction of the power of the beam during the propagation from $u = 0$ to $u \approx 23.2$ due to the scattering, it is strong enough to destroy the BH. This means that the inner and out horizons should meet and disappear. This process is seen in Fig. 1(b).

For this case, for high $v$, the BH is converted into an object where the outer $R$ and inner $R$-regions are connected. Now the test photons $u = const$ for all value of $u$ go to bigger $r$, when $v \to \infty$.

The mass function for big $v$ and big $u$ becomes greater than $m = 0.95$ but still the BH vanishes. It is related to the scattering of the exotic scalar field outside to bigger $r$. Of cause this is possible in the case of dynamical BHs. In this region (big $v$ and big $u$) we are at big $r$, definitely outside the (dynamical) BH which is at smaller $v$.

Fig. 3(b) represents the Penrose diagram that is confirmed by our numerical simulations for the cases when the BH was destroyed by the radiation.

At the end of this section we note the following. When the Reissner-Nordström BH is irradiated by a pulse of the exotic scalar radiation, the OAH becomes smaller (or disappears completely) and part of the outgoing radiation from the $T_-$ region can go to the outer $R'$-region in our Universe. This radiation may come into the $T_-$ region from the $R''$-region that belongs to another Universe, which is the counterpart of the outer $R'$-region of Fig. 3(b) in our Universe (from the left hand side of Fig. 3(b) outside the computational domain). This means that it is possible for some radiation from the other Universe to come to our $R'$-region. The propagation of the radiation in the opposite direction, from our $R'$-region to the $R''$-region in the other Universe, is still impossible. We call such an object a semi-traversable wormhole.
5 Conclusions

The processes arising when a Reissner-Nordström BH is irradiated by a beam of an exotic scalar field with a negative energy density have been analyzed. We performed the corresponding numerical computations using a numerical code specially designed for this purpose. It was demonstrated that these processes are quite different from the processes arising in the case of the irradiation of a Reissner-Nordström BH by a beam of a normal scalar field.

The evolution of the mass function demonstrates that in the case of the exotic scalar field, the evolution does not lead to the origin of a strong space-like singularity $r = 0$ in the $T$-region as was seen in the case of irradiation by the normal scalar field [2].

The numerical calculations demonstrate the manifestation of the antifocusing effects in the gravity field of an exotic scalar field with a negative energy density.

When the power of the exotic beam with a negative energy density is great enough, the mass function becomes less than the charge $q$ near the outer horizon. As a result, the BH disappears.

Again, we note that for full results and detailed analyses, see [1] in which we analyze a greater number of cases in detail (including the case of simultaneous normal and exotic scalar fields).

References