Towards detection of motion-induced radiation?

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Abstract

We discuss the results of theoretical simulations of a proposed experiment using pulsed laser irradiation of a semi-conductor diaphragm (SCD) in a superconducting microwave cavity [4]. Because the detection apparatus uses a single photon detection Rydberg beam method the analysis can be simplified to study the number of photons emitted in a given cavity mode.

1 Introduction & motivation

Motion-induced radiation, or the dynamical Casimir effect (DCE), has been a serious theoretical research subject for about the last 40 years and is important, because it challenges the principle of relativity of motion in vacuum and has analogs with quantum gravity, see Fig. 1. Various experimental proposals now suggest that the detection of this illusive radiation may be possible. We discuss the results of theoretical simulations of a proposed experiment that generates “effective motion” using pulsed laser irradiation of a semi-conductor diaphragm (SD).

Figure 1: A diagram discussing the analogy between quantum effects in electrodynamics and in curved spacetime [1].

We have has designed (PI: Seishi Matsuki) a DCE detection system, see Figure 2, using a GHz pulsed-laser of wavelength 860nm ($E_c = 1.55$eV) operating at a power $\sim 100\mu J$/pulse. The detection apparatus uses a single photon detection Rydberg beam and thus, the analysis can be simplified to study the number of photons emitted in a given cavity mode.

Questions?

• What is the best location, $\eta = d/L_2$, for the slab?
• Which frequencies dominate, dependant on pulse shape $n_s(t)$ (or relaxation time $t_r$)?
• Are there differences between TE and TM modes?

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**Figure 2:** A laser periodically irradiates a GaAs slab (~ 0.5mm thick) inside a superconducting (high-finesse: $Q \sim 10^6$) niobium cavity at ~ 100 mK. Note the single pulse duration is $T \sim 100$ps.

## 2 Plasma sheet model

The Hamiltonian for a surface plasma of electrons of charge $e$ and “effective mass” $m^*$ on a background electromagnetic field is

$$\mathcal{H} = \frac{1}{2} \int d^3x [\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}] + \int d^3x \left( \frac{1}{2m^*n_s} (\mathbf{p}_n - en_s \mathbf{A}_l)^2 + en_s \mathbf{A}_l \right) \delta (\mathbf{x} - \mathbf{x}_m)$$

(1)

where the momentum is $\mathbf{p}_n = (\mathbf{p}_z - en_s(t) \mathbf{A}_l)/m^*n_s(t)$, $n_s(t)$ is the “time dependent” surface charge density. Hamiltonian constraints imply $\mathbf{p}_z = 0$ [2] and thus, the electron momentum is related to the tangential vector potential by $\mathbf{p}_n = -eA_l/m^*$, which implies that the surface current density is $\mathbf{J} = en_s(t)\mathbf{E}_z = -e^2n_s(t)/m^* \mathbf{A}_l$. Using surface continuity [3]: $\mathbf{J} + n \cdot (\nabla \times \mathbf{E}) = 0$ with the Fitzgerald-Lorenz gauge condition: $\partial_t \Phi + \nabla \cdot \mathbf{A} = 0$ we arrive at

$$\dot{\mathbf{J}} = -e^2n_s(t)/m^* \nabla \times \mathbf{A}_l = e^2n_s(t)/m^* \partial_t A_0 \quad \Rightarrow \quad \sigma = e^2n_s(t)/m^* A_0,$$

(2)

where $A_0$ is the scalar potential. Applying the Hertz potentials $\Psi$ for TE and $\Phi$ for TM modes to separate Maxwell’s equations [4] we find the following jump conditions:

$$\nabla \Psi \parallel \Phi \partial_t^2 \Psi - \partial_t^2 \Psi = e^2n_s(t)/m^* \delta (z - d) \Psi (d), \quad \nabla \parallel \Phi \partial_t^2 \Phi - \partial_t^2 \Phi = 1 / k^2 \delta^2 (z - d) \Phi (d)$$

(3)

which can be derived from the wave equations below:

$$\nabla^2 \Psi + \partial_t^2 \Psi = e^2n_s(t)/m^* \delta (z - d) \Psi (d), \quad \nabla^2 \Phi + \partial_t^2 \Phi = 1 / k^2 \delta^2 (z - d) \Phi (d)$$

From continuity of the wave-function and jump conditions, for TE modes we have solutions:

$$\Psi_m = \begin{cases} 
A_m^{(\text{TE})} \sqrt{\frac{2}{L_z}} \sin(k_{m_z}z) \sqrt{\frac{2}{L_x}} \cos \left( \frac{\pi m_x}{L_x} \right) \sqrt{\frac{2}{L_y}} \cos \left( \frac{\pi m_y}{L_y} \right), & 0 \leq z < d \\
B_m^{(\text{TE})} \frac{1}{L_z} \sin \left( k_{m_z} (L_z - z) \right) \sqrt{\frac{2}{L_x}} \cos \left( \frac{\pi m_x}{L_x} \right) \sqrt{\frac{2}{L_y}} \cos \left( \frac{\pi m_y}{L_y} \right), & d \leq z < L_z
\end{cases}$$

(4)

and eigenvalue relation:

$$\sin(k_{m_z}L_z) / (k_{m_z})^{\mp 1} \sin (k_{m_z}(L_z - d)) \sin (k_{m_z}d) = \mp e^2n_s(t)/k^2 m^*$$

(5)

where the ± signs refer to TE and TM modes respectively and for TM replace $\sin \rightarrow \cos$ (for TE drop the $1/k^2$ factor).
3 Particle creation

The quantum field operator expansion \( \hat{\psi}(r,t) = \sum_m [a_m \psi_m(r,t) + a_m^\dagger \psi^*_m(r,t)] \) of the Hertz scalars with instantaneous basis ansatz (during irradiation):

\[
\psi_{\text{out}}^m(r,t) = \sum_m P_m^s \psi_m(r,t), \quad t \geq 0
\]

when substituted into the wave equations leads to

\[
\ddot{P}_n + \omega_n^2 P_n = -\sum_{m} \left[ 2M_{mn} \dot{P}_m^s + \dot{M}_{mn} P_m^s + \sum_{\ell} M_{mn} M_{\ell n} P^s_{\ell} \right]
\]

where \( \omega_m^2(t) = c^2 \left( \frac{m x L}{2} \right)^2 + \left( \frac{m y L}{2} \right)^2 + k_m^2 \) and \( M_{mn} = \langle \psi_n, \psi_m \rangle^{-1} \delta_{m,n} \delta_{m,n} \left( \frac{\partial \psi_m}{\partial t}, \psi_n \right) \) Note, the scalar product is defined by \( \langle \phi, \psi \rangle = -i \int_{\text{cavity}} d^3x \left( \phi \dot{\psi}^* - \dot{\phi} \psi^* \right) \). The Bogolubov coefficients are

\[
\alpha_{mn} = \langle \psi_{\text{out}}^m, \psi_{\text{in}}^n \rangle, \quad \beta_{mn} = -\langle \psi_{\text{in}}^m, [\psi_{\text{in}}^n]^* \rangle
\]

where in terms of the "instantaneous" mode functions

\[
\beta_{mn} = \sqrt{\frac{\omega_m}{2}} P_m^{(n)} - i \sqrt{\frac{1}{2\omega_m}} \left[ \dot{P}_m^{(n)} + \sum_{\ell} M_{\ell n} M_{\ell \ell} P_m^{(n)} \right]
\]

and \( \alpha_{mn} \) is obtained by complex conjugation. The number of photons in a given mode (for an initial vacuum state) is

\[
N_m(t) = \sum_n |\beta_{mn}|^2
\]

We vary \( \ell_{\text{max}} \sim 50 \) until the results do not change, but an independent check is the unitarity constraint: \( \sum_n (|\alpha_{mn}|^2 - |\beta_{mn}|^2) = 1 \), see Figure 3 (left inset).

4 Results & discussion

In Figure 3 (left) is a typical example of parametric enhancement for the lowest TE modes. On the right is the dependence on different slab locations, \( \eta \), for two different laser powers (unfilled triangles and circles correspond to 50\( \mu \)J/pulse or \( V_{\text{max}} L_z = 5000 \), while filled ones are for 0.01\( \mu \)J/pulse or \( V_{\text{max}} L_z = 1 \)).

Answers?

- The best location for the slab is \( \eta = 1/2 \) for TE modes.
- The fundamental frequencies dominate, where the pulse frequency is near to twice the parametric resonance frequency (besides de-tuning effects [4]).
- For TM modes even a low laser power generates significant photon production at \( \eta = 1/2 \). The TM case is fairly independent of \( \eta \) for large laser powers.

For future work it would be interesting to use a microscopic fermion model of a plasma sheet, like that used for Graphene [5]. This would allow one to consider Ohmic losses in the slab by coupling the microscopic Lagrangian to an external set of harmonic oscillators to model losses. Although Ohmic losses are mainly due to \( H_{\text{3g}} \), implying a strong TM dependence, it is still unclear if the losses in the slab affect TM modes more than TE modes.
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Figure 3: $N_{111}(t)$ fundamental mode. Left, TE with $\eta = 1/2$ and $V_{\text{max}} L_z = 5000$ with higher modes giving lesser contributions (inset shows unitarity constraint). Right, TE and TM mode dependence on $\eta$.

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