Beyond the Dirac-Nambu-Goto approximation in Brane-Black Hole systems

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Abstract

We consider curvature corrections to static, axisymmetric Dirac-Nambu-Goto membranes embedded into a spherically symmetric black hole spacetime with arbitrary number of dimensions. Since the next to leading order corrections in the effective brane action are quadratic in the brane thickness $\ell$, we adopt a linear perturbation approach in $\ell^2$. The perturbations are general in the sense that they are not restricted to the Rindler zone nor to the near-critical solutions of the unperturbed system. As a result, an unexpected asymmetry in the perturbed system is found. In configurations, where the brane does not cross the black hole horizon, the perturbative approach used here does not lead to regular solutions of the perturbation equation if the number of the brane’s spacetime dimensions $D > 3$. This condition, however, does not hold for the horizon crossing solutions. Consequently we argue that the perturbative approaches used here breaks down for subcritical type solutions near the axis of the system for $D > 3$. Nevertheless, we can discuss topology-changing phase transitions in cases when $D = 2$ or 3, i.e. when the brane is a 1-dimensional string or a 2-dimensional sheet, respectively. In the general case, a different approach should be sought. Based on the energy properties of those branes that are quasi-statically evolved from the equatorial configuration, we illustrate the results of the phase transition in the case of a $D = 3$ brane. It is found that small thickness perturbations do not modify the order of the transition, i.e. it remains first order just as in the case of vanishing thickness.

1 Notice

Higher dimensional black objects and branes are of importance and interest in several areas of present days physics. The classical black hole uniqueness theorems are known to fail in higher dimensions, and it turns out that a whole menagerie of black objects (black strings, rings, cigars, etc.) appear to exist. The study of new types of black objects became a very active research field recently, and among many other interesting aspects, the properties of possible transitions between the different types, or phases, is of special interest. For example, during the transition between a caged black hole and a black string phase, Kol demonstrated that the Euclidean topology of the system changes. This type of transition is called merger transition, and Kol found a strong similarity in its properties with the Choptuik critical collapse phenomena.

Recently, Frolov suggested a simple toy model with many features in common with merger and topology changing transitions. The model consists of a bulk $N$-dimensional black hole and a test $D$-dimensional brane in it $(D \leq N - 1)$, called brane-black hole (BBH) system. The black hole is spherically symmetric, static and can be neutral or charged. The brane is infinitely thin, and it is described by the Dirac-Nambu-Goto action. It is also static, spherically symmetric and it is assumed to reach asymptotic infinity in the form of a $(D - 1)$-dimensional plane. Due to the gravitational attraction of the black hole, the brane is deformed and there are two types of equilibrium configurations. The brane either

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1 This is a very brief summary of [1]. We refer the reader to the original reference for details and complete list of references.
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crosses the black hole horizon, or it lies totally outside of the black hole (see FIG. 1). In between the two types of configurations there exists a critical solution that separates the two phases. Frolov studied the transition between the so called subcritical phase (when the brane does not intersect the black hole horizon) and supercritical phase (when the brane crosses the horizon), and found a close similarity both with the merger transition in a caged black hole - black string system and with the Choptuik critical collapse phenomena.

The AdS/CFT correspondence also provides motivation to study the above BBH system. In fact, according to the correspondence, at sufficiently high temperature, a small number of flavors ($N_f$) of fundamental matter in strongly coupled gauge theories with a large number of colors ($N_c \gg N_f$), may be described, in the holographic dual, by probe Dq-branes in the gravitational background of a black hole. Using the tool of the gauge/gravity correspondence, Mateos et al. studied the phase transition of quark-antiquark bound states (mesons). They showed that in the case of an infinitely thin brane, the system generally undergoes a first order phase transition characterized by a change in the meson spectrum. The corresponding phase diagram in the vicinity of the critical solution exhibits a self similar structure, and this critical behavior and the first order transition are essentially universal to all Dp/Dq systems.

Importantly, it was pointed out, that higher order corrections to the brane effective action may cause, in principle, modifications to the above picture and it is likely that they spoil the system’s scaling symmetry and self-similar behavior. Indeed, higher-derivative corrections to the D-brane action correspond to finite ’t Hooft coupling corrections in the holographic dual, and provide a more realistic description of the system. These corrections may become important in the vicinity of the phase transition, since the curvature of the brane becomes large there.

In the context of low-scale gravity theories, the possibility that a micro black hole may form in high energy collisions, like those at the LHC, re-creates a setup similar to the BBH system described above. In particular, the question whether a black hole may escape into the extra dimensional bulk has raised some attention, due to the potential phenomenological relevance. Clarifying the role of the thickness of the brane in that context is also an important issue.

The dynamics of branes keeps also attracting attention in the context of higher dimensional generalizations of the Bernstein conjecture and the study of the stability of brane-black hole systems.

For all the above reasons, it is important to go beyond the approximation of zero thickness and consider higher order, curvature corrections coming from small thickness perturbations in the BBH system. Curvature corrections to the dynamics of domain walls without self-gravitation, in the case of non-zero thickness have been investigated earlier by Carter and Gregory. They demonstrated that the next to leading order contribution is quadratic in the wall width (the brane thickness) and they obtained an exact, analytic expression for the corresponding effective action in terms of the intrinsic Ricci scalar $R$ and the extrinsic curvature scalar $K$.

In the present paper we consider thickness perturbations to the BBH system using the general curvature corrected effective brane action of Carter and Gregory. To analyze this system we followed a perturbative treatment, in the sense that we treated the curvature corrections as small perturbations in the effective action (they are indeed very small as being quadratic in the perturbation parameter). To obtain the dynamical equation for the perturbations, we used the quadratic perturbation parameter to expand the 4th-order Euler-Lagrange equation and kept the linear terms only. As a result, a second order, linear equation was found to describe the perturbations, with a very complicated source term. We analyzed the asymptotics of the perturbation equation, and found that there is no subcritical solution regular on the axis of the system above a certain dimension. This implies that our perturbation method is not appropriate in this region. We argued that the full non-perturbative solution may cure the problem. Although we do not report them in this brief summary, we have presented the analytic solution for far distances and the numerical solution in the near horizon region of the perturbation equation for various dimensions. We also addressed the question of the phase transition in the case of a $D = 3$ dimensional brane. For this purpose we have used the approach of Flachi et al. based on the energy properties of a quasi-static brane evolution from the equatorial configuration.

Small thickness perturbations to the brane dynamics are derived from higher-order, curvature corrections to the effective action of the brane. In the present model we do not consider the self-gravitation of the brane, hence the curvature scalars are completely determined by the embedding black hole spacetime. In the approximation when all the relevant dynamical length scales $L$ of the system are very large com-
pared to the parameter $\ell$ that characterizes the thickness of the brane, the BBH system can be described by the following exact, analytic expression for the effective action:

$$S = \int d^D \zeta \sqrt{-\det g_{\mu\nu}} \left[ -\frac{8\mu^2}{3\ell^2} (1 + C_1 R + C_2 K^2) \right],$$

(1)

where $R$ is the Ricci scalar, $K$ is the extrinsic curvature scalar and the coefficients $C_1$ and $C_2$ are expressed by the wall thickness parameter as $C_1 = (\pi^2 - 6)\ell^2/24$ and $C_2 = -\ell^2/3$. The parameter $\mu$ is related to the thickness as $\ell = (\mu \sqrt{2\lambda})^{-1}$ which originates from a field theoretical domain-wall model, where $\mu$ is the mass parameter and $\lambda$ is the coupling constant of the scalar field. The details of the Euler-Lagrange equation are very lengthy and will be omitted here. In brief, we studied thickness perturbations to static, $D$-dimensional, Dirac-Nambu-Goto branes embedded into higher dimensional, spherically symmetric, black hole spacetimes. The perturbations originate from higher order, curvature corrections added to the thin brane action, and are quadratic in the thickness of the brane. Applying a linear perturbation method with the perturbation parameter $\ell^2/L^2$, we derived the general form of the perturbation equation for a brane that is axisymmetric and has a form of a $(D - 1)$-dimensional plane at asymptotic infinity.

From the analysis of the asymptotic behavior of the perturbation equation, we found that there is no regular solution of the perturbed problem in the Minkowski embedding case, unless the brane is a string, or a sheet. This restriction, however does not hold for the black hole embedding solutions, which are always regular within our perturbative approach. The $D = 3$ case should not be too different from branes with larger dimensionality as long as additional symmetries are imposed on the brane angular directions.

From this result, we concluded that the absence of regular solutions above the dimension $D = 3$ implies, that the problem can not be solved within our perturbative approach around the thin solution, which is not smooth on the axis of the system. Hence, for a general discussion, one needs to find a new, exact solution of the curvature corrected problem, that is expected to behave differently from the thin solution and it is smooth on the axis. After the above conclusions, we provided the solution of the perturbation equation for various brane $(D)$ and bulk $(N)$ dimensions. The far distance equations are integrated analytically, while the near horizon solutions are obtained by numerical computations. The deformations of the perturbed brane configurations are plotted and a comparison is made with the corresponding thin brane configurations with identical boundary conditions, for both types of solution. A sample result is illustrated in Fig. 1, where the thick (red) brane configurations are shown together with their thin (blue) counterparts in a cylindrical coordinate system, in the case of an $N = 5, n = 2$ black hole embedding.

Figure 1: The picture shows the thick (red) brane configurations together with their thin (blue) counterparts in a cylindrical coordinate system, in the case of an $N = 5, n = 2$ black hole embedding. The initial conditions are $\theta_0 = \frac{\pi}{2}$ (bottom curves) and $\frac{\pi}{2} - \pi$ (top curves), and the thickness parameter $\ell$ is chosen to be large for the purpose of making the effects visible. The black curve represents the black hole’s event horizon.

One motivation of this paper was to consider the effects of higher derivative, curvature corrections on the first order phase transition between the Minkowski and black hole branch, that is present in the unperturbed system. With the solution of the perturbed problem we found that within a perturbative
approach, one can consider a phase transition between the two branches only in the cases of $D = 2$ or 3. We investigated the properties of this transition in the case of a $D = 3$ brane, and found that small thickness perturbations do not modify the qualitative behavior of the phase transition, i.e. it remains a first order one, just as in the case of zero thickness.

Since our perturbative approach does not provide a regular thick brane solution for dimensions $D > 3$, we cannot answer in the most general way the question whether higher order, curvature corrections in the effective brane action can change the order of the phase transition in the BBH system or not. Although we expect that small corrections may not change the picture too much, as they are quadratic in the thickness of the brane, a definitive answer can only be given within a non-perturbative approach. Further study to address this question is in progress.

2 References

References