Subfactors and topological invariants of 3-manifolds

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I will discuss some symmetry on the finite dimensional non-degenerate commuting squares and would like to explain how I have been inspired by Ocneanu's topological quantum field theory(TQFT) in three dimensions to describe symmetry on those kind of commuting squares.

A basic tool to construct inclusions of hyperfinite II_1 factors with finite Jones index would be the method of the commuting squares. From a finite dimensional non-degenerate commuting square

$$\begin{array}{ccc} R_{00} & \subset & R_{01} \\ \cap & & \cap \\ R_{10} & \subset & R_{11} \end{array}$$

we can make the double sequence of finite dimensional C^*-algebras $\{R_{ij}\}$ by iterating the Jones basic construction. Then we get horizontal and vertical inclusions of hyperfinite II_1 factors with finite Jones indices $N = \vee_{n=0}^{\infty} R_{0n} \subset \vee_{n=0}^{\infty} R_{1n} = M$ and $P = \vee_{n=0}^{\infty} R_{n0} \subset \vee_{n=0}^{\infty} R_{n1} = Q$ under the assumption that Bratteli diagrams of the initial inclusions in the commuting square are connected.

A natural question is whether there is any relationship between those two inclusions. In particular, when one of those inclusions is of finite depth, then is it true that so is the other? Such questions are first raised by V. Jones in 1995. This question was answered affirmatively using techniques from paragroup theory and the answer says those inclusions have the same "size as C^*-tensor categories".

When we have an inclusion of hyperfinite II_1 factors with finite depth(or C^*-tensor category with finitely many irreducible objects), we can construct the topological quantum field theory based on the triangulation of the given 3-dimensional manifold V due to Ocneanu. In the case of a closed manifold, we get a complex number. The simplest example will be the three dimensional sphere S^3. In this case, the value of the theory is given by 1/(the size of C^*-tensor category). With this observation and the above relationship between the vertical and horizontal inclusions, we will see the generalization of the relationship to TQFT's.

Extracting certain property of TQFT constructed from an inclusion of hyperfinite II_1 factors with finite index and finite depth, we will finally arrive at the notion of "equivalence" of the inclusions, which tells us that equivalent inclusions have equivalent "representation theory" to each other. We will see there always exists some symmetry described by equivalence on the commuting squares in question.